UNIVERSITY OF TARTU

1632

Hackathon

Workshop-2

Date : 30.09.2023

Properties of Fourier Transform

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CIPHR

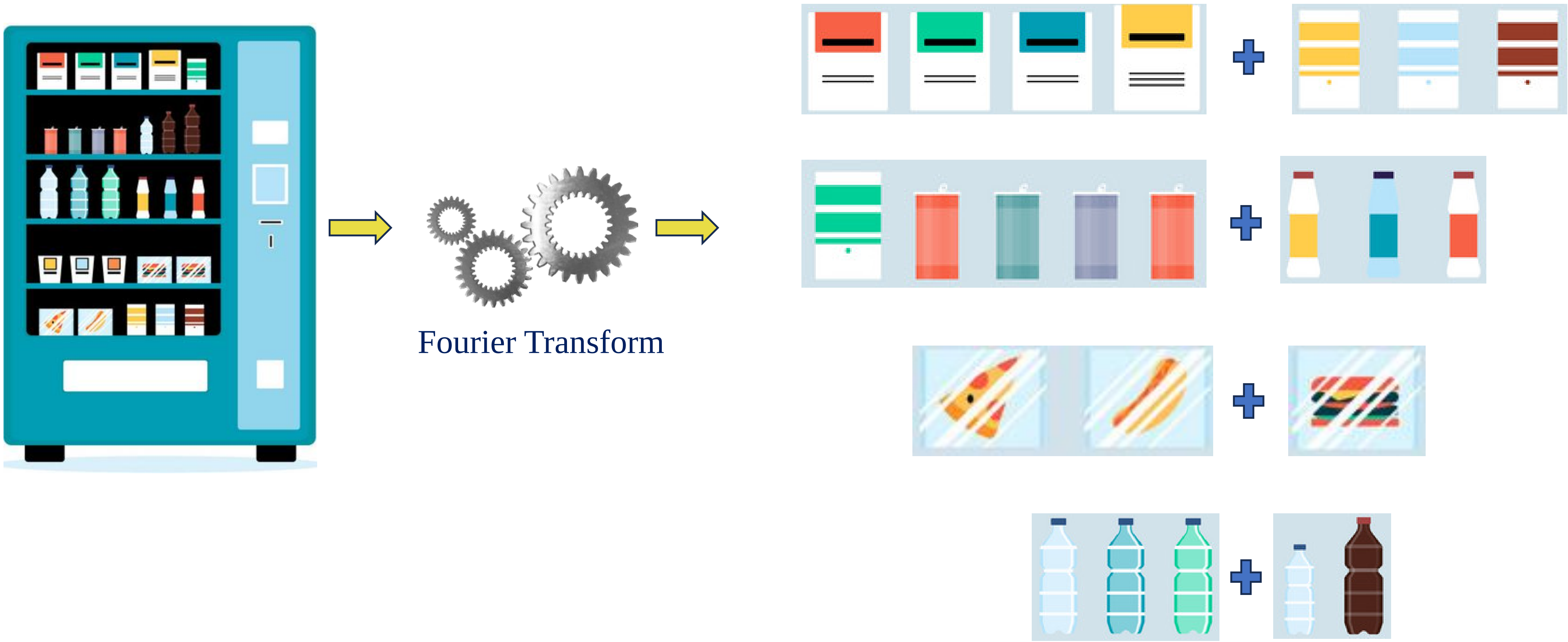
This Project has received funding from the European Union' s Horizon 2020 research and innovation programme under grant agreement No 857627 (CIPHR)

Outline of presentation

- Linearity theorem
- Similarity theorem
- Shift theorem
- Parseval — Rayleigh theorem
- Tasks

CIPHR

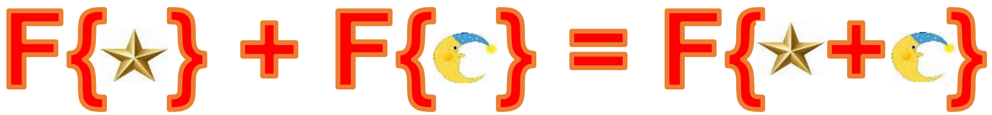
Fourier Transform

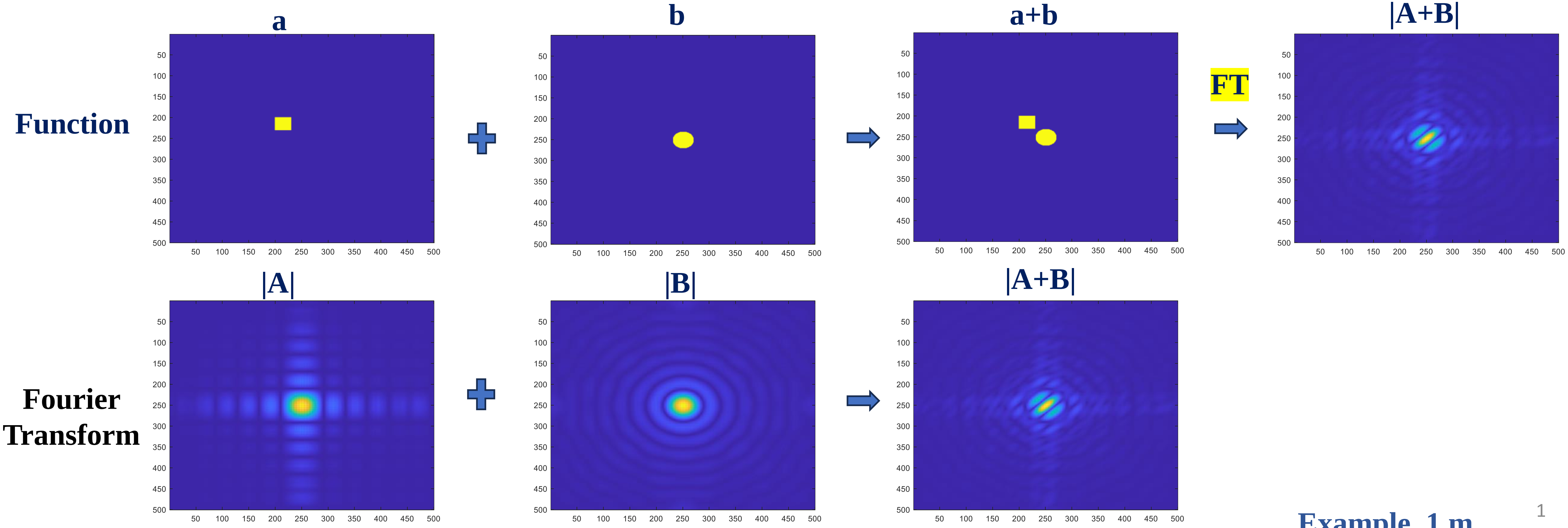


The information is the same in both domains but only differently represented.

Linearity Theorem

Linearity theorem. $\mathcal{F}\{\alpha g + \beta h\} = \alpha\mathcal{F}\{g\} + \beta\mathcal{F}\{h\}$; that is, the transform of a weighted sum of two (or more) functions is simply the identically weighted sum of their individual transforms.





Example_1.m

Linearity Theorem

Calibration:

```
%% Calibration
clear; %clears workspace
N=500; % define matrix size
x=-N/2:N/2-1; % creates x co-ordinate axis
y=-N/2:N/2-1; % creates y co-ordinate axis
pixel=10*10^-6; % define pixel size
lambda=0.6328*10^-6; % define wavelength
[X,Y]=meshgrid(x*pixel,y*pixel); % creates meshgrid for x & y values
R=sqrt(X.*X+Y.*Y); % define radius
Aperture=zeros(N,N); % define a matrix with zeros
Aperture(R<N/2*pixel)=1; % creates aperture
imagesc(Aperture) % display aperture
```

Linearity Theorem

The FT of a sum is the sum of the FT's:

```
%% Square aperture
C1=zeros(N,N);
C1(200:230,200:230)=1;
figure, imagesc(C1);
colormap parula
%% Circular aperture
C2=zeros(N,N);
C2(R<20*pixel)=1;
imagesc(C2);colormap parula
C=C1+C2;
Figure;
imagesc(C); colormap parula
```

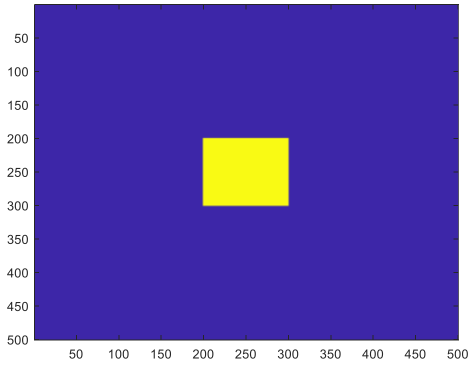
```
%% Fourier Transform
f=0.2;
Lens=exp(-1i*(pi/(lambda*f))*(X.*X+Y.*Y));
Q=exp(1i*(pi/(lambda*f))*(X.*X+Y.*Y));
D=ifftshift(ifft2(fft2(C).*fft2(Q)));
E=D.*Lens;
F1=ifftshift(ifft2(fft2(E).*fft2(Q)));
I=F1.*conj(F1);
F1=F1/max(max(F1));
figure ; imagesc(abs(F1))
```

Similarity Theorem

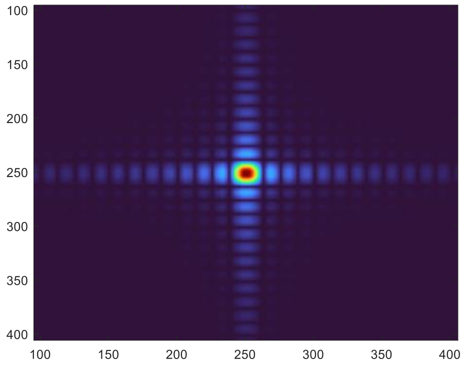
That is a "stretch" of the coordinates in the space domain (x,y) results in a contraction of the coordinates in the frequency domain (fx,fy), plus a change in the overall amplitude of the spectrum.

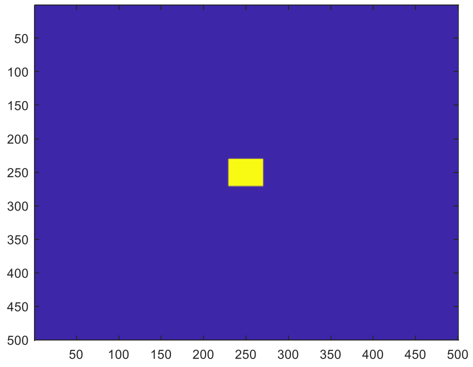
Signal

Fourier Transform

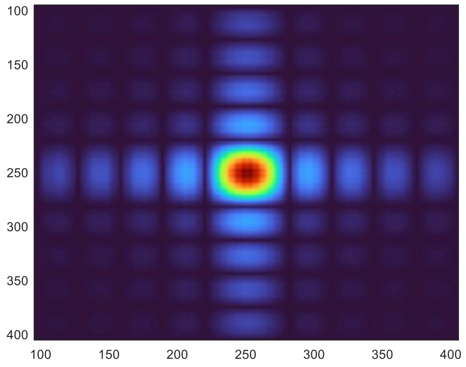


FT





FT



Space domain

Frequency domain

Example_2.m

Similarity Theorem

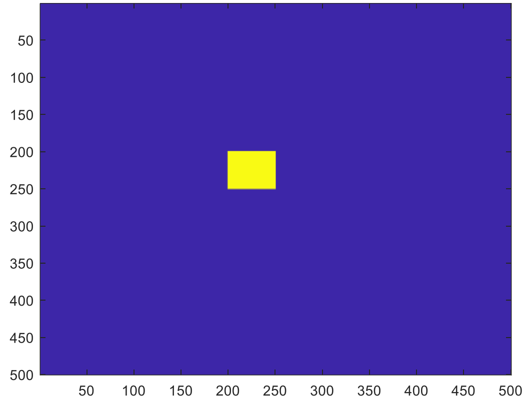
```
%% Square aperture
C1=zeros(N,N);
C1(200:300,200:300)=1;
figure, imagesc(C1);
colormap parula

%% Small square aperture
C1=zeros(N,N);
C1(230:270,230:270)=1;
imagesc(C1);colormap parula
```

```
%% Fourier Transform
f=0.2;
Lens=exp(-1i*(pi/(lambda*f))*(X.*X+Y.*Y));
Q=exp(1i*(pi/(lambda*f))*(X.*X+Y.*Y));
D=ifftshift(ifft2(fft2(C).*fft2(Q)));
E=D.*Lens;
F=ifftshift(ifft2(fft2(E).*fft2(Q)));
I=F.*conj(F);
figure ; colormap turbo
imagesc(abs(F))
```

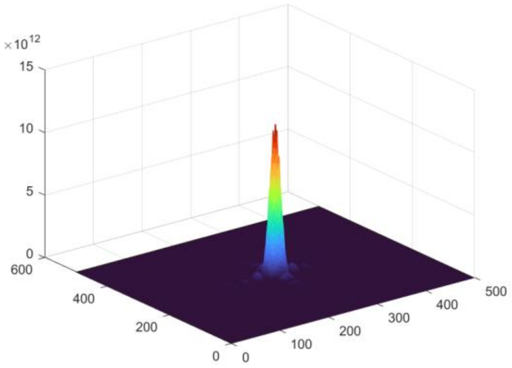
Shift Theorem

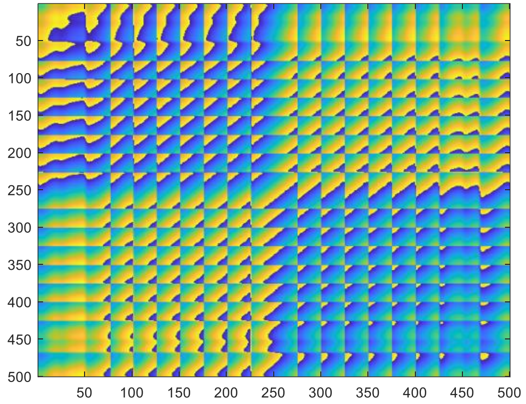
That is, translation in the space domain introduces a linear phase shift in the frequency domain.

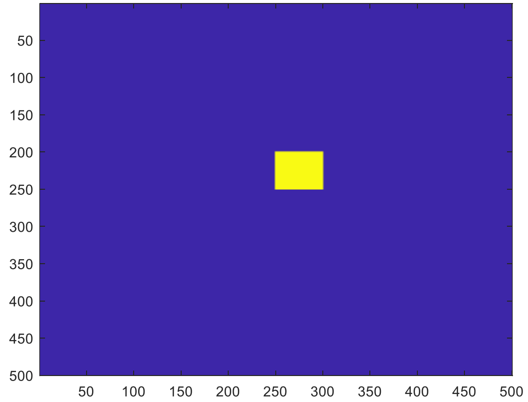
Signal | Intensity | Phase



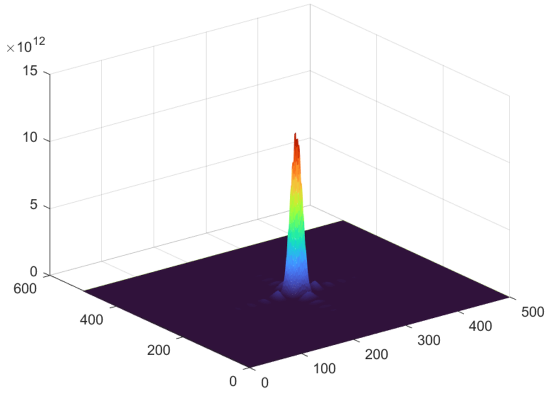
Fourier Transform

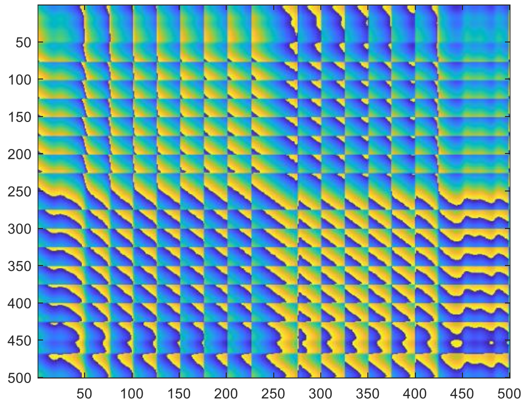






Fourier Transform





Example_3.m

Shift Theorem

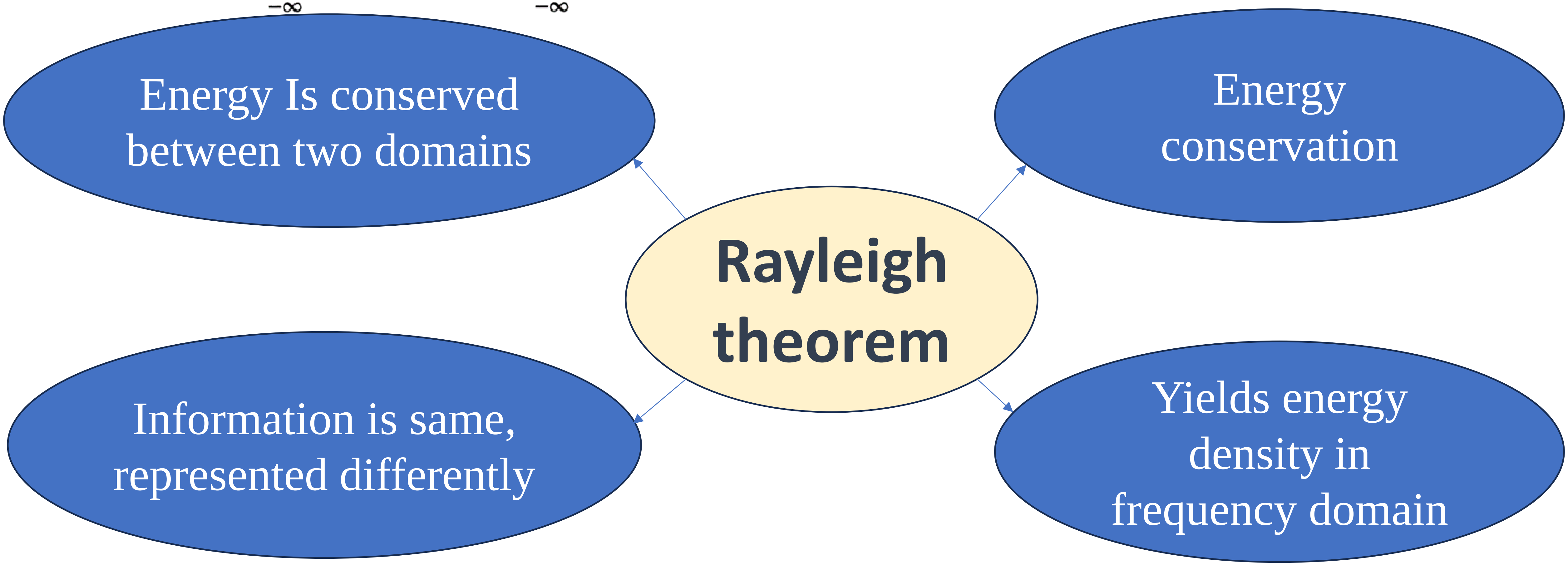
```
%% Square aperture
C1=zeros(N,N);
C1(200:250,200:250)=1;
figure, imagesc(C1);
colormap parula

%% Shifted square aperture
C1=zeros(N,N);
C1(200:250,250:300)=1;
imagesc(C1);colormap parula
```

```
%% Fourier Transform
f=0.2;
Lens=exp(-1i*(pi/(lambda*f))*(X.*X+Y.*Y));
Q=exp(1i*(pi/(lambda*f))*(X.*X+Y.*Y));
D=ifftshift(ifft2(fft2(C1).*fft2(Q)));
E=D.*Lens;
F=ifftshift(ifft2(fft2(E).*fft2(Q)));
I=F.*conj(F);
figure ; colormap parula
imagesc(abs(F))
figure ; colormap parula
imagesc(angle(F))
Plot(I(251,: ))
meshz(I); colormap parula
```

Rayleigh Theorem

Rayleigh theorem, $\mathcal{F}\{g(x, y)\} = G(f_X, f_Y)$

$$\iint_{-\infty}^{\infty} |g(x, y)|^2 \, dx \, dy = \iint_{-\infty}^{\infty} |G(f_X, f_Y)|^2 \, df_X \, df_Y .$$



Tasks

1. Show similarity theorem for any Picture of your interest.
2. Choose any two objects and show minimum and maximum fringe width.

Thank you

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